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NOTE ON SYLOW'S THEOREM

BY G. A. MILLER

SYLOW'S theorem is of such fundamental importance that any improvement in stating it is of interest. We desire to suggest the following form:

THEOREM. *If the order of a group* (G) *is* $p^{\alpha}m$, *p being any prime which does not divide m, then G contains at least one subgroup* (P_{α}) *of order* p^{α}. *Moreover, if P_{α} contains only one subgroup* (P_{β}) *of a given type, then all the subgroups of this type in G are conjugate and their number is of the form* $1 + kp$. *The order of G may be written in the form* $p^{\beta}n(1 + kp)$, $p^{\beta}n$ *being the order of the largest subgroup which transforms P_{β} into itself.*

The first sentence of this theorem does not differ materially from the one usually given. The rest is more general than the usual statement. It reduces to this when β has the special value α. It gives more information about G than the usual form does only when something is known about P_{α}. For example, when P_{α} is cyclic, it follows that all the subgroups whose order is a given power of p form a single conjugate set and that the number of these subgroups is $1 + kp$. The additional information conveyed in these cases seems to justify the slight increase in the length of the theorem.

If we assume Sylow's theorem (in its usual form) proved, the more general statement given above follows very easily. Such a subgroup of order p^{β} could not transform another subgroup of this type into itself, otherwise P_{α} would include more than one such subgroup of order p^{β}. From this it follows exactly as in the case for p^{α} that the number of these subgroups of order p^{β} is of the form $1 + kp$ and that they form a single conjugate set.

STANFORD UNIVERSITY,
OCTOBER, 1903.